

THE HANLE EFFECT IN 1D, 2D AND 3D

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Abstract. This paper¹ addresses the problem of scattering line polarization and the Hanle effect in one-dimensional (1D), two-dimensional (2D) and three-dimensional (3D) media for the case of a two-level model atom without lower-level polarization and assuming complete frequency redistribution. The theoretical framework chosen for its formulation is the QED theory of Landi Degl’Innocenti (1983), which specifies the excitation state of the atoms in terms of the irreducible tensor components of the atomic density matrix. The self-consistent values of these density-matrix elements is to be determined by solving jointly the kinetic and radiative transfer equations for the Stokes parameters. We show how to achieve this by generalizing to Non-LTE polarization transfer the Jacobi-based ALI method of Olson *et al.* (1986) and the iterative schemes based on Gauss-Seidel iteration of Trujillo Bueno and Fabiani Bendicho (1995). These methods essentially maintain the simplicity of the Λ -iteration method, but their convergence rate is extremely high. Finally, some 1D and 2D model calculations are presented that illustrate the effect of horizontal atmospheric inhomogeneities on magnetic and non-magnetic resonance line polarization signals.

1. Introduction

The scattering line polarization, and its modification due to a weak magnetic field —such that the Zeeman splitting is negligible compared with the line width (the so called Hanle effect; Hanle, 1924)—, sensitively depends on the *anisotropy* of the radiation field and on the magnetic field vector geometry (Landi Degl’Innocenti, 1985; Stenflo, 1994). The solar atmospheric plasma is spatially inhomogeneous, with vertical and horizontal variations, not only in temperature, macroscopic velocity and density, but also in the orientation and intensity of the magnetic field (see Sánchez Almeida, 1999 for new insights in this respect). It is thus clear that, in order to fully exploit the Hanle effect as a diagnostic tool for weak magnetic fields, we also

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need to address the problem of resonance line polarization and the Hanle effect in 2D and 3D media where the radiation field's anisotropy is different from that corresponding to the currently-assumed 1D atmospheric models.

To this end, this contribution begins presenting a formulation of resonance line polarization and the Hanle effect that we consider as the most suitable one for practical RT applications. It is based on the density-matrix theory for the generation and transfer of polarized radiation (see Landi Degl'Innocenti, 1983; 1984; 1985). In this paper we consider the standard case of a two-level model atom neglecting atomic polarization in its lower level (i.e. it is assumed that the lower-level Zeeman sublevels are equally populated and that there are no coherences among them). The quantities whose *self-consistent* values are to be determined are the irreducible tensor components of the density matrix (ρ_Q^K), which depend only on the spatial coordinates. The statistical equilibrium (SE) and RT equations to be solved are valid independently of whether we assume 1D, 2D or 3D geometries.

A summary of previous work done in the subject of the numerical solution of Non-LTE polarization transfer problems can be found in Trujillo Bueno and Manso Sainz (1999). In this respect, we should mention the recent work of Nagendra *et al.* (1998; see also their contribution in these proceedings) where the Hanle effect in 1D is considered using a different theoretical approach. For information concerning the numerical solution of more general polarization transfer problems formulated with the density-matrix theory see Trujillo Bueno (1999).

The outline of this paper is as follows. Section 2 presents the basic equations for the case of a two level atom without lower-level atomic polarization and assuming complete frequency redistribution. Section 3 shows how the very efficient iterative methods of solution investigated by Trujillo Bueno and Fabiani Bendicho (1995) (Jacobi, Gauss-Seidel and successive over-relaxation) can be suitably generalized to the problem of Non-LTE polarized radiative transfer in the Hanle effect regime. Finally, in Sect. 4 we present the results of some illustrative 2D Hanle-effect calculations for triplet lines and discuss the ensuing horizontal transfer effects.

2. Basic Equations

In scattering line polarization and Hanle effect problems, the quantum interferences (or coherences) between the magnetic sublevels of each atomic level must be considered. In order to properly take into account these effects, we work within the framework of the polarization transfer theory based on the irreducible tensor components of the atomic density matrix (see Bommier and Sahal Br  chot, 1978; Landi Degl'Innocenti, 1983). Since we are restricting ourselves in this paper to the standard case of a two-level

model atom neglecting lower-level atomic polarization only the $K = Q = 0$ *irreducible tensor component* of the density matrix suffices to completely describe the lower-level excitation. For instance, if the lower level of total population N_l has angular momentum $J_l = 0$ one has that $\rho_0^0(l) = N_l/\sqrt{3}$.

However, with respect to the upper level we need to specify its total population ($\rho_0^0(u)$), its alignment ($\rho_0^2(u)$), and two complex quantities ($\rho_1^2(u)$ and $\rho_2^2(u)$) which take into account the coherences between the sublevels of the upper level. For example, for an upper level with $J_u = 1$

$$\rho_0^0(u) = \frac{1}{\sqrt{3}}(N_1 + N_0 + N_{-1}) = \frac{1}{\sqrt{3}}N_u, \quad (1)$$

$$\rho_0^2(u) = \frac{1}{\sqrt{6}}(N_1 - 2N_0 + N_{-1}), \quad (2)$$

$$\rho_1^2(u) = -\frac{1}{\sqrt{2}}[\rho_{J_u}(1, 0) - \rho_{J_u}(0, -1)], \quad (3)$$

$$\rho_2^2(u) = \rho_{J_u}(1, -1), \quad (4)$$

where N_i ($i = 1, 0, -1$) are the populations of the magnetic sublevels $M = 0, \pm 1$ of the upper level, and $\rho_J(M, M') = \langle \alpha J M | \rho | \alpha J M' \rangle$ (with $|\alpha J M \rangle$ the eigenvectors of the atomic Hamiltonian) are the elements of the density matrix in the standard representation (see, e.g., Messiah, 1969). We point out that the *orientation* components (ρ_Q^1) of the density matrix are zero because we are assuming a static medium for which the radiation field that illuminates its boundaries has no circular polarization (see Landi Degl'Innocenti *et al.*, 1990).

For each irreducible *upper* level tensor component ρ_Q^K with $Q > 0$, there exists another spherical component with $Q < 0$ related to it through the conjugation property:

$$\rho_{-Q}^K = (-1)^Q [\rho_Q^K]^*, \quad (5)$$

where the symbol “*” means complex conjugation. We can thus choose the following linear combinations as independent variables:

$$\tilde{\rho}_Q^K = \frac{1}{2}[\rho_Q^K + (-1)^Q \rho_{-Q}^K] = \text{Re}[\rho_Q^K], \quad Q > 0, \quad (6)$$

$$\hat{\rho}_Q^K = \frac{1}{2i}[\rho_Q^K - (-1)^Q \rho_{-Q}^K] = \text{Im}[\rho_Q^K], \quad Q > 0, \quad (7)$$

where “ i ” is the imaginary unit. Finally, normalizing all these *upper* level *unknowns* to the total atomic population of the lower level, we find the six *real* unknowns of this problem: ρ_0^0 , ρ_0^2 , $\tilde{\rho}_1^2$, $\hat{\rho}_1^2$, $\tilde{\rho}_2^2$ and $\hat{\rho}_2^2$ of the upper level.

In the QED polarization transfer theory of Landi Degl'Innocenti (1983, 1984), the radiation field is described by its spherical tensor components:

$$J_0^0 = \int dx \phi_x \oint \frac{d\Omega}{4\pi} I_{x\Omega} \quad (8)$$

$$J_0^2 = \int dx \phi_x \oint \frac{d\Omega}{4\pi} \frac{1}{2\sqrt{2}} [(3\mu^2 - 1)I_{x\Omega} + 3(\mu^2 - 1)Q_{x\Omega}] \quad (9)$$

$$J_1^2 = \int dx \phi_x \oint \frac{d\Omega}{4\pi} \frac{\sqrt{3}}{2} e^{i\chi} \sqrt{1 - \mu^2} [-\mu(I_{x\Omega} + Q_{x\Omega}) - iU_{x\Omega}] \quad (10)$$

$$J_2^2 = \int dx \phi_x \oint \frac{d\Omega}{4\pi} \frac{\sqrt{3}}{2} e^{2i\chi} [\frac{1}{2}(1 - \mu^2)I_{x\Omega} - \frac{1}{2}(1 + \mu^2)Q_{x\Omega} - i\mu U_{x\Omega}] \quad (11)$$

where $I_{x\Omega}$, $Q_{x\Omega}$ and $U_{x\Omega}$ are the Stokes parameters relative to the direction Ω specified by the angles θ and χ defined as in Fig. (2b), $\mu = \cos \theta$, and ϕ_x is the line profile, with x the frequency measured from the line center in units of the Doppler width. The physical meaning of these expressions is quite simple. Note that J_0^0 is the well-known frequency integrated mean intensity. The other three irreducible tensor components of the radiation field are frequency and angular integrals of the three Stokes parameters, weighted by some angle-dependent quantities, and by the line profile ϕ_x . The radiation field tensor J_0^2 measures the degree of vertical-horizontal anisotropy: it is positive when the radiation is predominantly vertical, negative if horizontal, and it vanishes at the bottom of the atmosphere where the radiation field is unpolarized and isotropic. Finally, the two other spherical tensor components are complex, and they measure the breaking of the axial symmetry of the radiation field through the azimuthal exponentials appearing inside the angular integrals. Therefore, they are zero in axially-symmetric media like 1D plane-parallel atmospheres with a vertical magnetic field, or without any field at all. The J_Q^K components with $Q < 0$ can be obtained through a conjugation relation similar to the one stated in Eq. (5) for the atomic statistical tensors. With equivalent definitions to those of Eqs. (6)-(7), we obtain the six real quantities J_0^0 , J_0^2 , \tilde{J}_1^2 , \hat{J}_1^2 , \tilde{J}_2^2 and \hat{J}_2^2 .

The SE equations that govern the six unknowns of this problem are:

$$[1 + \delta^{(K)}(1 - \epsilon)] \begin{pmatrix} S_0^0 \\ S_0^2 \\ \tilde{S}_1^2 \\ \hat{S}_1^2 \\ \tilde{S}_2^2 \\ \hat{S}_2^2 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & M_{11} & M_{12} & M_{13} & M_{14} & M_{15} \\ 0 & M_{21} & M_{22} & M_{23} & M_{24} & M_{25} \\ 0 & M_{31} & M_{32} & M_{33} & M_{34} & M_{35} \\ 0 & M_{41} & M_{42} & M_{43} & M_{44} & M_{45} \\ 0 & M_{51} & M_{52} & M_{53} & M_{54} & M_{55} \end{pmatrix} \begin{pmatrix} S_0^0 \\ S_0^2 \\ \tilde{S}_1^2 \\ \hat{S}_1^2 \\ \tilde{S}_2^2 \\ \hat{S}_2^2 \end{pmatrix} + (1 - \epsilon) w_{J_u J_l}^{(K)} \begin{pmatrix} J_0^0 \\ J_0^2 \\ \tilde{J}_1^2 \\ -\hat{J}_1^2 \\ \tilde{J}_2^2 \\ -\hat{J}_2^2 \end{pmatrix} + \epsilon \begin{pmatrix} B_{\nu_{ul}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad (12)$$

where $S_Q^K = (2h\nu_{ul}^3/c^2) [(2J_l + 1)/\sqrt{2J_u + 1}] \rho_Q^K$.

The M_{ij} -quantities of the *magnetic operator* \mathcal{M} are coefficients that depend on the strength and orientation of the local magnetic field (see Table 1 in Landi Degl’Innocenti *et al.*, 1990; for their explicit values), $w_{J_u J_l}^{(K)}$ is a numerical factor depending on the total angular momentum of the levels involved in the transition (see Table I in Landi Degl’Innocenti, 1984; and note that it is unity for a $J_l = 0$ and $J_u = 1$ line transition), ϵ is the collisional destruction probability due to inelastic collisions, and $B_{\nu_{ul}}$ is the Planck function. We point out that $\delta^{(K)}$ is the collisional depolarizing rate due to elastic collisions measured in units of the Einstein A_{ul} coefficient, with $\delta^{(0)} = 0$ in the first equation.

These SE equations have a clear physical meaning. The magnetic operator \mathcal{M} couples the $K = 2$ statistical tensors among them. This is a local term because its M_{ij} coefficients only depend on the local value of the magnetic field. The second term in the *r.h.s.* of Eq. (12) is the radiative coupling term. It couples the atomic system with the radiation field and therefore it is highly non-local. Finally, the third term is the unpolarized thermal source.

Since, as mentioned above, the orientation components are zero only three Stokes parameters are relevant in this problem: $I_{x\Omega}$, $Q_{x\Omega}$ and $U_{x\Omega}$. Due to the fact that the lower level is assumed to be unpolarized, the radiative transfer equation for each Stokes parameter is decoupled from the others:

$$\frac{d}{d\tau_x} \mathcal{Z}_{x\Omega} = \mathcal{Z}_{x\Omega} - \mathcal{S}_{\mathcal{Z}}, \quad (13)$$

with $\mathcal{Z}_{x\Omega}$ the Stokes parameter $I_{x\Omega}$, $Q_{x\Omega}$ or $U_{x\Omega}$, and $d\tau_x = -(\chi_l \phi_x + \chi_c) ds$ (with s the geometrical distance along the ray path and $\chi_{l,c}$ the line-integrated and continuum opacities). Although our code is very general, and takes into account the effect of a background continuum, for notational simplicity we will not consider it explicitly in the following equations. Thus, the line contributions to the source functions components $\mathcal{S}_{\mathcal{Z}}$ are:

$$S_I^{line} = S_0^0 + w_{J_u J_l}^{(2)} \left\{ \frac{1}{2\sqrt{2}} (3\mu^2 - 1) S_0^2 - \sqrt{3}\mu \sqrt{1 - \mu^2} (\cos \chi \tilde{S}_1^2 - \sin \chi \hat{S}_1^2) \right. \\ \left. + \frac{\sqrt{3}}{2} (1 - \mu^2) (\cos 2\chi \tilde{S}_2^2 - \sin 2\chi \hat{S}_2^2) \right\}, \quad (14)$$

$$S_Q^{line} = w_{J_u J_l}^{(2)} \left\{ \frac{3}{2\sqrt{2}} (\mu^2 - 1) S_0^2 - \sqrt{3}\mu \sqrt{1 - \mu^2} (\cos \chi \tilde{S}_1^2 - \sin \chi \hat{S}_1^2) \right. \\ \left. - \frac{\sqrt{3}}{2} (1 + \mu^2) (\cos 2\chi \tilde{S}_2^2 - \sin 2\chi \hat{S}_2^2) \right\}, \quad (15)$$

$$S_U^{line} = w_{J_u J_l}^{(2)} \sqrt{3} \left\{ \sqrt{1 - \mu^2} (\sin \chi \tilde{S}_1^2 + \cos \chi \hat{S}_1^2) + \mu (\sin 2\chi \tilde{S}_2^2 + \cos 2\chi \hat{S}_2^2) \right\}, \quad (16)$$

where the S_Q^K -quantities are given in terms of the density-matrix elements ρ_Q^K as indicated by the expression given after Eq. (12).

Since the transfer equations (13) are decoupled, it is straightforward to write the formal solution for $I_{x\Omega}$, $Q_{x\Omega}$ and $U_{x\Omega}$, as in the standard unpolarized case, through the monochromatic $\Lambda_{\nu\Omega}$ operator (see Mihalas, 1978). Thus, with the values of the density matrix elements we calculate S_I , S_Q and S_U , and then any suitable formal solution method of the standard RT equation can be used to calculate the Stokes parameters $I_{x\Omega}$, $Q_{x\Omega}$ and $U_{x\Omega}$ at each grid point of the chosen spatial grid of NP points, for all the frequencies and directions of the chosen numerical quadrature. This allows us to write the radiation field tensors in the absence of a background continuum as:

$$\begin{pmatrix} \mathbf{J}_0^0 \\ \mathbf{J}_0^2 \\ \tilde{\mathbf{J}}_1^2 \\ -\hat{\mathbf{J}}_1^2 \\ \tilde{\mathbf{J}}_2^2 \\ -\hat{\mathbf{J}}_2^2 \end{pmatrix} = \begin{pmatrix} \Lambda_{00} & \Lambda_{01} & \Lambda_{02} & \Lambda_{03} & \Lambda_{04} & \Lambda_{05} \\ \Lambda_{10} & \Lambda_{11} & \Lambda_{12} & \Lambda_{13} & \Lambda_{14} & \Lambda_{15} \\ \Lambda_{20} & \Lambda_{21} & \Lambda_{22} & \Lambda_{23} & \Lambda_{24} & \Lambda_{25} \\ \Lambda_{30} & \Lambda_{31} & \Lambda_{32} & \Lambda_{33} & \Lambda_{34} & \Lambda_{35} \\ \Lambda_{40} & \Lambda_{41} & \Lambda_{42} & \Lambda_{43} & \Lambda_{44} & \Lambda_{45} \\ \Lambda_{50} & \Lambda_{51} & \Lambda_{52} & \Lambda_{53} & \Lambda_{54} & \Lambda_{55} \end{pmatrix} \begin{pmatrix} \mathbf{S}_0^0 \\ \mathbf{S}_0^2 \\ \tilde{\mathbf{S}}_1^2 \\ \hat{\mathbf{S}}_1^2 \\ \tilde{\mathbf{S}}_2^2 \\ \hat{\mathbf{S}}_2^2 \end{pmatrix} + \begin{pmatrix} \mathbf{T}_0^0 \\ \mathbf{T}_0^2 \\ \tilde{\mathbf{T}}_1^2 \\ -\hat{\mathbf{T}}_1^2 \\ \tilde{\mathbf{T}}_2^2 \\ -\hat{\mathbf{T}}_2^2 \end{pmatrix}, \quad (17)$$

where the NP \times NP operators $\Lambda_{\alpha\beta}$ are frequency and angular weighted averages of the standard $\Lambda_{\nu\Omega}$ operator, and \mathbf{J}_Q^K , \mathbf{S}_Q^K and \mathbf{T}_Q^K are vectors of length NP, with the \mathbf{T}_Q^K components given by expressions similar to Eqs. (8)-(11) but using the transmitted Stokes parameters due to the incident radiation at the boundaries instead of $I_{x\Omega}$, $Q_{x\Omega}$ and $U_{x\Omega}$. The analytic expressions of these Λ -like operators can be found in Manso Sainz & Trujillo Bueno (in preparation). As shown below, the only operator of relevance for our iterative approach is Λ_{00} , which is nothing but the $\bar{\Lambda}$ operator of the standard unpolarized case:

$$\Lambda_{00}(i, j) = \frac{1}{4\pi} \int \phi_x dx \oint d\Omega \Lambda_{\nu\Omega}(i, j). \quad (18)$$

It can be demonstrated that in plane-parallel atmospheres all the non-diagonal operators are zero except $\Lambda_{01} = \Lambda_{10}$ (see also Landi Degl'Innocenti *et al.*, 1990; Nagendra, Frisch & Faurobert-Scholl, 1998), and that in 2D media Λ_{03} , Λ_{05} , Λ_{13} , Λ_{15} , Λ_{23} , Λ_{25} , Λ_{34} , Λ_{45} and their symmetric ones are zero (Manso Sainz & Trujillo Bueno, in preparation).

3. Iterative methods of solution

The most simple iterative scheme to solve this coupled set of equations is the Λ -iteration method. Given an estimate S_Q^{Kold} of the unknowns (i.e. of the S_Q^K tensors at all the spatial grid-points) solve formally the transfer equations to calculate the corresponding six J_Q^{Kold} radiation field tensors at each spatial grid-point “ i ”. Then, introduce these $J_Q^{Kold}(i)$ values into the SE equations at each spatial grid-point independently and get an improved *new* set of S_Q^K -values. As shown by Trujillo Bueno and Manso Sainz (1999), this Λ -iteration method can be used as a reliable solution method *if* one initializes with the self-consistent ρ_0^0 -values corresponding to the case in which polarization phenomena are neglected.

In order to develop iterative methods characterized by an extremely high convergence rate, and that can be applied to find the self-consistent solution independently of the chosen initialization, we need to account *implicitly* for some of the “new” values of the unknowns ρ_Q^K -elements. To this end, in the following we generalize to the Hanle effect regime the Jacobi-based ALI method of Olson *et al.* (1986) and the iterative methods of Trujillo Bueno and Fabiani Bendicho (1995) that are based on Gauss-Seidel iteration (see also Trujillo Bueno and Manso Sainz, 1999).

In order to derive a Jacobi-based ALI scheme we do the same as with the Λ -iteration method, except that in order to calculate $J_0^0(i)$ we use the *new* value of S_0^0 (instead of the *old* one) at the grid point “ i ” being considered. Since this “*new*” value is not yet known, we implicitly write it *only* in the expression of J_0^0 , i.e. we write

$$\begin{aligned} J_0^0(i) &\approx J_0^{0old}(i) + \Lambda_{00}(i, i) \delta S_0^0(i), \\ J_Q^2(i) &\approx J_Q^{2old}(i), \end{aligned} \quad (19)$$

where $\delta S_0^0(i) = S_0^{0new}(i) - S_0^{0old}(i)$. After substitution of these expressions for the $J_Q^K(i)$ quantities into the SE equations, we obtain at each grid-point “ i ” independently a system of six equations with six unknowns that can be solved easily to find the *new* values of the six statistical tensors $S_Q^K(i)$. We point out that this is equivalent to applying the operator splitting technique to the Λ_{00} operator only. It can be demonstrated that no gain is obtained if the splitting is applied to the whole set of 36 operators of Eq. (17) (see next subsection below).

We emphasize that, at each iterative step, the δS_Q^K corrections are made *point by point*. Thus, a better idea than Jacobi’s method would be to apply the method based on Gauss-Seidel (GS) iteration of Trujillo Bueno and Fabiani Bendicho (1995), i.e. to do the same as with the Λ -iteration method, except that in order to calculate $J_0^0(i)$ we use the *new* values of S_0^0 (instead

of the *old* ones) at the grid point “ i ” being considered and *also* at the grid-points $1, 2, \dots, i-1$ that have been *previously* considered as we advance from the atmosphere’s boundary at which the first correction was made. Since $S_0^{new}(i)$ is not yet known, we implicitly write it *only* in the expression of $J_0^0(i)$, i.e. we write

$$\begin{aligned} J_0^0(i) &\approx J_0^{oldandnew}(i) + \Lambda_{00}(i, i) \delta S_0^0(i), \\ J_Q^2(i) &\approx J_Q^{2old}(i), \end{aligned} \quad (20)$$

where $J_0^{oldandnew}(i)$ is the averaged mean intensity calculated using the “*new*” values of S_0^0 at the grid-points $1, 2, \dots, i-1$ and the “*old*” S_Q^K -values at points $i, i+1, i+2, \dots, \text{NP}$, with NP the total number of points of the spatial grid (see Trujillo Bueno and Fabiani Bendicho, 1995; for details).

After substitution of these expressions for the $J_Q^K(i)$ quantities into the SE equations, we obtain at each grid-point “ i ” a system of six equations with six unknowns that can be easily solved to find the *new* values of the statistical tensors $S_Q^K(i)$. By implementing this GS-based iterative scheme as suggested in the conclusions of the paper by Trujillo Bueno and Fabiani Bendicho (1995) the total computational work required to achieve the self-consistent solution is a factor 4 smaller than with the previous Jacobi-based method.

An extra important improvement of the convergence rate can be achieved by multiplying each δS_Q^K GS-correction by a numerical factor (ω) lying between 1 and 2. This is the successive over-relaxation (SOR) method, i.e.

$$\delta S_Q^{K\text{SOR}} = \omega \delta S_Q^{K\text{GS}} \quad (21)$$

The optimal value of ω can be easily found (see Trujillo Bueno & Fabiani Bendicho 1995); however, a good convergence rate can be obtained by just choosing $\omega = 1.5$.

Figure 1 shows the convergence rates of the six S_Q^K unknowns, i.e. it shows the variation with the iteration number of the maximum relative change (R_c) in these quantities. The problem considered was the Hanle effect in a 2D model atmosphere with $\epsilon = 10^{-4}$, $\delta^{(2)} = 0$ and with a non-vertical magnetic field vector. The solid lines refer to the Jacobi-based ALI method. We point out that the S_Q^2 tensor elements converge at the same rate as S_0^0 . The reason for this is that all the J_Q^K radiation field tensors are dominated by the Stokes I parameter, and this specific intensity is basically set by the value of S_0^0 (see Trujillo Bueno and Manso Sainz, 1999). Furthermore, the convergence rate of S_0^0 is the same in polarized (with or without magnetic fields) and in unpolarized problems as shown by the dotted line in Fig. 1 that cannot be distinguished from the S_0^0 solid-line (see also Faurobert-Scholl *et al.*, 1997). It is important to point out that this is

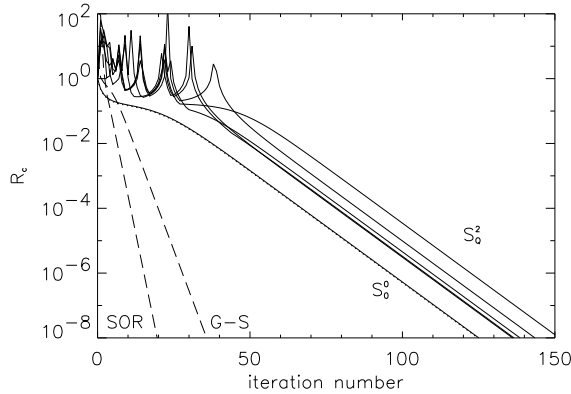


Figure 1. The convergence rate of the six S_Q^K unknowns for the Jacobi method (solid lines), and of S_0^0 for the Gauss-Seidel and the SOR method with $\omega = 1.5$ (dashed lines). See the text for the meaning of the dotted line.

a general behaviour and it is independent of the geometry of the medium (1D, 2D or 3D).

The convergence rates for the Gauss-Seidel and SOR iterative methods are also plotted in Fig.1. For clarity reasons the convergence rates for the S_Q^2 elements have been omitted because they present a similar behaviour. As seen in the figure our GS method is four times faster than Jacobi, while our SOR method would be a factor 10 if the calculation had been performed with the optimal ω -value (see also Trujillo Bueno and Manso Sainz, 1999).

4. Some illustrative examples

In the following we show some illustrative examples of the Hanle effect in 2D atmospheric models comparing the results with the corresponding 1D case. Although our code is very general and can deal with realistic 2D and 3D scenarios with horizontal periodic boundary conditions, here we will restrict ourselves to sinusoidal fluctuations of the Planck function along the horizontal X-axis:

$$B_\nu = \bar{B}_\nu(z) + \Delta B_\nu \cos(k_x x), \quad (22)$$

with $k_x = 2\pi/L$ the horizontal wavenumber, and L the horizontal wavelength of the thermal inhomogeneities. All the geometrical distances are measured in units of the opacity scale height ($\mathcal{H}_\chi \simeq 100$ km). We assume a gaussian line absorption profile and consider different k_x -values, being $k_x = 0$ the plane-parallel 1D limit.

Figure 2 is presented to facilitate the understanding of the geometry of the problem and to indicate the chosen positive and negative directions of

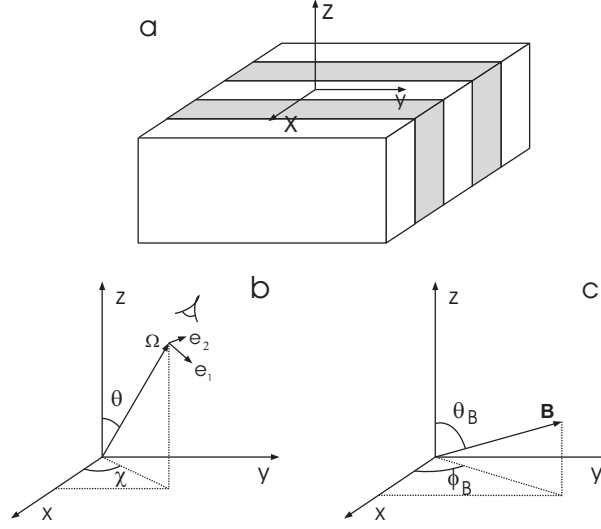


Figure 2. (a) Schematic geometry of a 2D atmosphere with a sinusoidally varying temperature inhomogeneity. (b) Show the angles defining the line-of-sight (Ω) and the polarization unit vectors \mathbf{e}_1 and \mathbf{e}_2 . The positive $Q_{x\Omega}$ direction is along \mathbf{e}_1 . (c) Angles defining the magnetic field orientation.

the Stokes Q-parameter. In Fig. 2a the shaded and white “slabs” simply aim at visualizing the coolest and hottest regions of the assumed 2D model atmosphere, respectively. Note, however, that the chosen 2D medium is not composed of such embedded slabs, since it is characterized by the horizontal temperature fluctuations given by the previous equation. These “slabs” should be understood as infinite along the Y-direction. Fig. 2b shows the angles that specify the direction of propagation of the ray under consideration, while Fig. 2c gives the angles that determine the orientation of the magnetic field vector. As seen in Fig. 2a the Y-Z plane at $X=0$ is dividing the hottest “slabs” in two equal halves. We will show the emergent polarization profiles for simulated observations made along the hottest “slabs” (i.e. at the horizontal position $X=0$ and for a line of sight with $\chi = 90^\circ$). Thus, the positive direction of the Stokes Q-parameter lies along the slabs, and the negative one is perpendicular to them.

In Fig. 3 we consider the case of resonance line polarization in this 2D atmosphere. When observing at disk center (i.e. at $\mu = \cos\theta = 1$) we find that there is no polarization signal corresponding to the plane-parallel 1D limit (i.e. for $k_x = 0$), as it must indeed be the case because the radiation field of an unmagnetized 1D medium has axial symmetry, and the radiation field tensors \bar{J}_1^2 and \bar{J}_2^2 are then zero. As we increase the horizontal wavenumber k_x (i.e. the parameter that measures the degree of horizontal

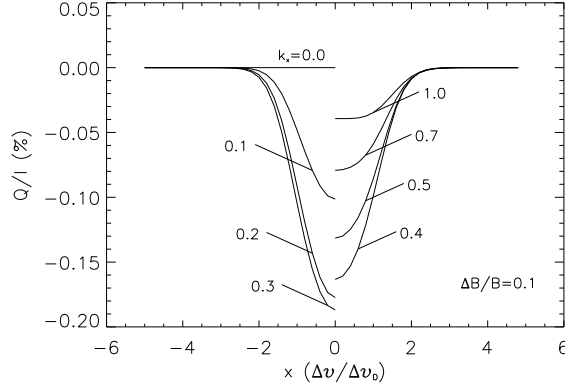


Figure 3. Emergent Q/I profiles at disk center ($\mu = 1$) for different horizontal wavenumber k_x -values. Here $\Delta B_\nu / \bar{B}_\nu = 0.1$.

inhomogeneity of our 2D model) the radiation field loses its axial symmetry, and the polarization signal Q/I starts to increase accordingly. However, when the horizontal temperature inhomogeneities are smaller than about 2000 km (i.e. for wavenumbers $k_x > 0.3$), the polarization signal decreases as the wavenumber k_x further increases. This is because we start then to approach the limiting case of an atmosphere composed of optically thin irregularities, for which the radiation field recovers the axial symmetry characteristic of a 1D medium.

Fig. 4 shows the emergent Q/I and U/I profiles for simulated high-spatial resolution observations with a line of sight having $\chi = 90^\circ$ and made close to the solar limb ($\mu = 0.1$) in a 2D atmosphere with $\Delta B_\nu / \bar{B}_\nu = 0.1$ and $k_x = 0.3$ (i.e. with $L \approx 2000$ km). The magnetic field direction is defined through the azimuthal and polar angles ϕ_B and θ_B (see Fig. 2c), and its strength through the parameter $\Gamma = 0.88 g_J B / A_{ul}$, with g_J the upper level Landé factor, B the magnetic field in Gauss, and A_{ul} the spontaneous emission Einstein coefficient measured in units of $10^7 s^{-1}$. Since the line-of-sight lies along the slabs (i.e. $\chi = 90^\circ$), we find that in the absence of magnetic field (i.e. for $\Gamma = 0$) U/I is zero due to symmetry reasons. The two uppermost panels show how a magnetic field parallel to the slab (i.e. with $\phi_B = \theta_B = 90^\circ$) substantially changes the emergent polarization signal. However, when the magnetic field is perpendicular to the slab (see the lowermost panels with $\theta_B = 90^\circ$ and $\phi_B = 0^\circ$), the change with Γ of Q/I is the smallest one and U/I always remains zero. The two central panels show the intermediate case with $\phi_B = 45^\circ$. The dashed lines show the results for the plane-parallel $k_x = 0$ case.

Fig. 5 shows the emergent fractional linear polarization Q/I at $\mu = 0.1$ and $\chi = 90^\circ$, when the Planck function varies sinusoidally (with $k_x = 0.3$

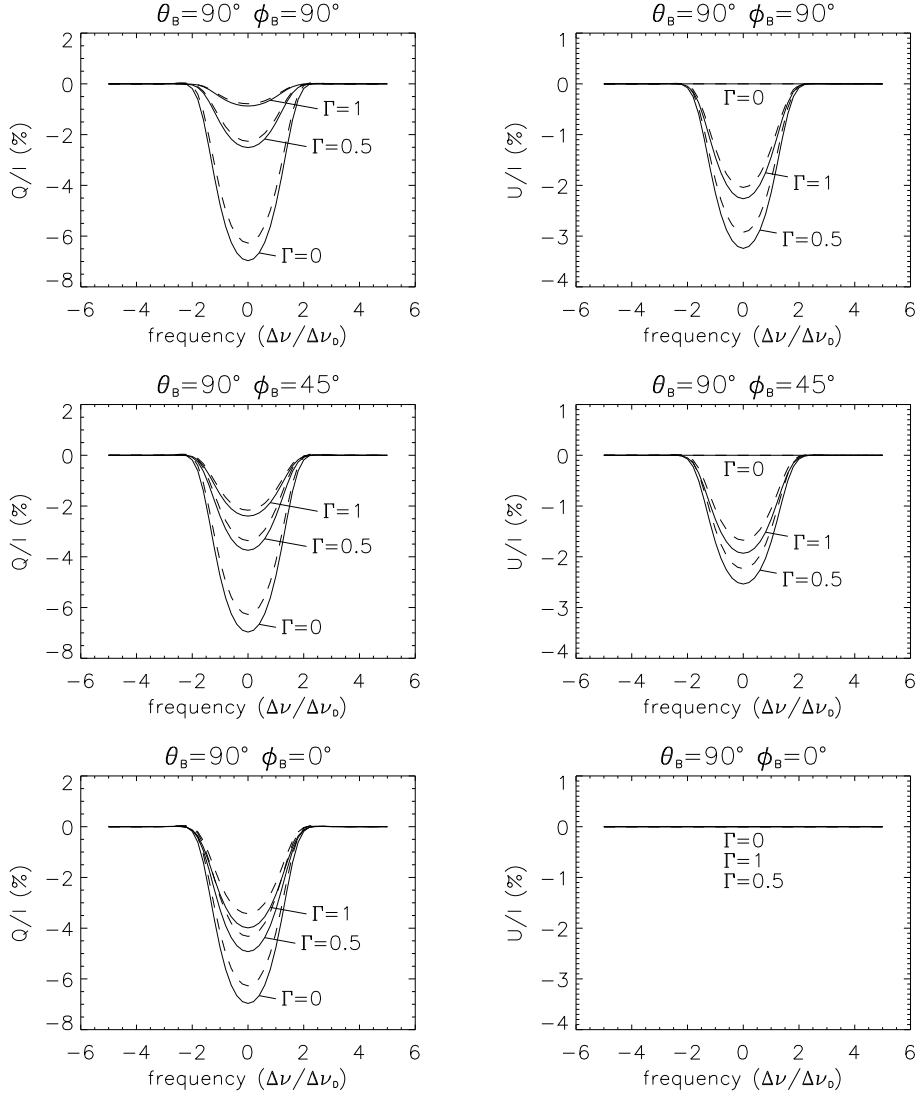


Figure 4. Emergent Q/I and U/I profiles in the assumed 2D atmosphere (solid lines) with wavenumber $k_x = 0.3$ and $\Delta B_\nu/\bar{B}_\nu = 0.1$. The line of sight observation is at $\mu = 0.1$ and $\chi = 90^\circ$. Dashed lines indicate the 1D case.

and $\Delta B_\nu/\bar{B}_\nu = 0.2$), and there exists additionally a horizontal fluctuation in the line opacity χ_l that is anticorrelated with that of the Planck function, i.e.

$$\chi_l = \bar{\chi}_l[1 + \alpha \cos(k_x x)], \quad (23)$$

with $k_x = 0.3$ and $\alpha = -0.2$. The curves situated on the left-hand-side of the figure show the 1D and 2D results corresponding to the zero magnetic

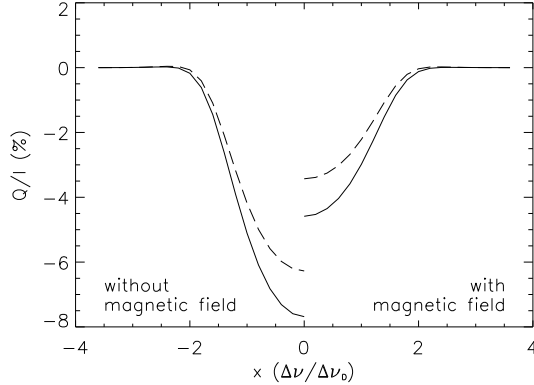


Figure 5. As Fig. 4 but taking $\Delta B_\nu/\bar{B}_\nu = 0.2$ and including the horizontal opacity inhomogeneities given by Eq. (23). The magnetic field has $\Gamma = 1$, $\theta_B = 90^\circ$, and $\phi = 0^\circ$. Solid lines show the 2D case, and dashed-lines the 1D one.

field case, while the curves on the *r.h.s.* correspond to a case of a magnetic field with $\Gamma = 1$, $\theta_B = 90^\circ$, and $\phi_B = 0^\circ$. For this magnetic and line-of-sight geometry $U/I = 0$. It can be seen that, for this particular model atmosphere and geometry, a Hanle-effect diagnostic of very high spatial resolution observations would lead to an underestimation the magnetic field strength for interpretations based on the plane-parallel 1D approximation.

5. Conclusions

We have developed a Hanle effect code that allows the numerical simulation of resonance line polarization signals in the presence of weak magnetic fields in 1D, 2D and 3D media. The governing equations have been formulated working within the framework of the density matrix polarization transfer theory of Landi Degl’Innocenti (1983, 1985). These SE and RT equations are the same independently of whether we are considering 1D, 2D or 3D atmospheric models. The six ρ_Q^K -unknowns of the problem are neither frequency nor angle dependent, since they only vary with the spatial position.

Three different iterative schemes that were originally developed for RT applications in the unpolarized case have been generalized to solve this set of equations: Jacobi (ALI), Gauss-Seidel and SOR (Trujillo Bueno & Fabiani Bendicho, 1995, Trujillo Bueno & Manso Sainz 1999). This kind of iterative methods does not make use of any matrix inversion, and essentially maintain the Λ -iteration simplicity. The only difference between the 1D, 2D and 3D versions of our Hanle effect code lies in the formal solution routine that calculates the radiation field tensors from the current values of the density-matrix elements. To this end, in 2D we use the formal solver

developed by Auer, Fabiani Bendicho and Trujillo Bueno (1994) and in 3D we use the one presented at this workshop by Fabiani Bendicho and Trujillo Bueno (1999).

We have also shown some Hanle effect results for 1D and 2D media. These calculations illustrate how weak magnetic fields and horizontal radiative transfer effects compete to modify the scattering line polarization signals expected from plane-parallel 1D atmospheres. Thus, further careful investigations must be done in order to separate both effects, when diagnosing weak solar magnetic fields via the Hanle effect. This type of future studies should be done thinking in the interpretation of *low* spatial resolution scattering line polarization observations. Our numerical approach is very efficient and suitable to investigate scattering polarization signals for a variety of atmospheric models having any desired temperature, density and magnetic field vector variations. Another useful research that can be done with our Hanle effect codes concerns the simulation of polarization signals emerging from realistic MHD and semi-empirical 2D and 3D models.

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